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MODEL OF NONLINEAR EVOLUTION OF LONG-WAVE PERTURBATIONS ON AN IDEALLY CONDUCTING JET WITH CURRENT IN A LONGITUDINAL MAGNETIC FIELD. COLLISION OF MAGNETIZED JETS

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Within the framework of the magnetohydrodynamic approach, a system of equations is derived for nonlinear evolution of long-wave axisymmetric perturbations on a conducting fluid jet with surface electric current, located along the axis of a conducting solid cylinder in a longitudinal magnetic field. The fluid is assumed to be inviscid, incompressible, and ideally conducting, like the cylinder walls. It is shown that, if the longitudinal field is uniform and the axial flow is shear-free, this system can be either hyperbolic or elliptic-hyperbolic, depending on problem parameters. The boundaries of hyperbolicity and ellipticity regions in the space of solutions are determined. In the hyperbolicity region, equations of characteristics and conditions on them are obtained. The problem of the decay of velocity discontinuity on the jet is considered. Conditions are found for the existence of a continuous self-similar solution in the hyperbolicity region, corresponding to collision of jets.

Key words: *magnetic hydrodynamics, jet, long-wave approximation.*

Introduction. Analytical studies of evolution of perturbations on fluid conductors with free boundaries have been performed in the linear approximation and mainly by spectral methods [1–3]. The direct Lyapunov method could be recently applied to these problems [4]. Still, there are insufficient analytical studies of the nonlinear stage of perturbation evolution.

In studying nonlinear problems, because of their complexity, various approximate models that describe essential features of the processes considered are frequently used. One of such simplifications is the long-wave or shallow-water asymptotic approximation used in studying waves in the fluid [5, 6]. Within the framework of shallow-water models, it became possible to study important features of nonlinear effects typical of the flows considered, develop an exact theory, and also solve applied problems. In addition, this theory was mathematically justified by studying the flow of a uniform fluid in a thin layer [7, 8].

In the present paper, the long-wave approximation is extended to the case of an MHD jet flow with a free boundary. A model is proposed, which describes the nonlinear behavior of long-wave perturbations on a conducting fluid jet with surface electric current in a longitudinal magnetic field. This model allows one to perform analytical studies and has a certain physical meaning, which is confirmed by results obtained for a uniform longitudinal magnetic field and shear-free axial flow.

1. Formulation of the Problem. We study a conducting fluid jet of unlimited length in a longitudinal magnetic field with a constant electric current J passing on the jet surface. The jet is located along the axis of an infinitely conducting cylinder of radius r_0. We introduce a cylindrical coordinate system (r^*, φ, z^*); the z^* axis coincides with the jet centerline. The following notation is used: v_1, v_2, v_3, H_1, H_2, H_3, H_1^*, H_2^*, and H_3^* are the fluid-velocity components and magnetic field inside and outside the jet corresponding to the coordinate system (r^*, φ, z^*), P is the pressure, ρ is the density, and t^* is the time. It is assumed that $v_2 \equiv 0$ and $H_2 \equiv 0$ during

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fluid motion in the conducting jet. In addition, it is assumed that this motion is axisymmetric and the fluid itself is inviscid, incompressible, and ideally conducting. The action of surface-tension forces at the free boundary of the jet is ignored.

Based on these assumption, the equations of one-fluid ideal magnetic hydrodynamics [9] take the form

$$\rho\Big(\frac{\partial v_1}{\partial t^*}+v_1\,\frac{\partial v_1}{\partial r^*}+v_3\,\frac{\partial v_1}{\partial z^*}\Big)=-\frac{\partial P_*}{\partial r^*}+\frac{H_1}{4\pi}\,\frac{\partial H_1}{\partial r^*}+\frac{H_3}{4\pi}\,\frac{\partial H_1}{\partial z^*},$$

$$\rho\Big(\frac{\partial v_3}{\partial t^*}+v_1\,\frac{\partial v_3}{\partial r^*}+v_3\,\frac{\partial v_3}{\partial z^*}\Big)=-\frac{\partial P_*}{\partial z^*}+\frac{H_1}{4\pi}\,\frac{\partial H_3}{\partial r^*}+\frac{H_3}{4\pi}\,\frac{\partial H_3}{\partial r^*},$$

$$\frac{\partial(Ar^*)}{\partial t^*}+v_1\,\frac{\partial(Ar^*)}{\partial r^*}+v_3\,\frac{\partial(Ar^*)}{\partial z^*}=0, \tag{1.1}$$

$$H_1=-\frac{\partial A}{\partial z^*},\qquad H_3=\frac{1}{r^*}\,\frac{\partial(Ar^*)}{\partial r^*},$$

$$\frac{1}{r^*}\,\frac{\partial(v_1r^*)}{\partial r^*}+\frac{\partial v_3}{\partial z^*}=0.$$

Here $P_*\equiv P+(H_1^2+H_3^2)/(8\pi)$ is the modified pressure and A is the azimuthal component of the vector potential (magnetic permeability of the conducting jet is assumed to be equal to unity).

If the displacement current is neglected, the equations of the magnetic field outside the jet are

$$\frac{\partial H_1^*}{\partial z^*}-\frac{\partial H_3^*}{\partial r^*}=0,\qquad H_2^*=\frac{2J}{r^*},\qquad \frac{\partial H_3^*}{\partial z^*}+\frac{1}{r^*}\,\frac{\partial(H_1^*r^*)}{\partial r^*}=0. \tag{1.2}$$

The following boundary conditions are imposed at the axis of the conducting jet, its boundary $[r^*=r_1(z^*,t^*)]$, and cylinder walls:

$$v_1=0,\qquad H_1=0\qquad (r^*=0),$$

$$P_*=\frac{(H_1^*)^2+(H_2^*)^2+(H_3^*)^2}{8\pi},\quad v_1=\frac{\partial r_1}{\partial t^*}+v_3\,\frac{\partial r_1}{\partial z^*}\qquad (r^*=r_1(z^*,t^*)),$$

$$H_1-H_3\,\frac{\partial r_1}{\partial z^*}=0,\qquad H_1^*-H_3^*\,\frac{\partial r_1}{\partial z^*}=0\qquad (r^*=r_1(z^*,t^*)), \tag{1.3}$$

$$H_1^*=0\qquad (r^*=r_0).$$

In passing to the long-wave approximation, we introduce the dimensionless variables and quantities t, η, z, q, w, p_*, h, H, a, h^*, æ, and H^*:

$$r^{*2}=\eta L^2\delta^2,\quad z^*=zL,\quad t^*=tL/v_0,\quad 2v_1r^*=qv_0L\delta^2,\quad v_3=wv_0,\quad P_*=p_*\rho v_0^2,$$

$$2H_1r^*=hL\delta^2H_0,\quad H_3=HH_0,\quad 2Ar^*=a\delta^2L^2H_0,$$

$$2H_1^*r^*=h^*L\delta^2H_0,\quad H_2^*r^*=æL\delta H_0,\quad H_3^*=H^*H_0.$$

Here L is the characteristic scale along the z^* axis, H_0 is the characteristic value of the magnetic field equal to H_2^* for $r^*=r_{10}$ $(H_0=2J/r_{10})$, r_{10} is the characteristic radius of the jet, $v_0=H_0/(4\pi\rho)^{1/2}$ is the characteristic velocity, and $\delta=r_{10}/L$. It is assumed that $\delta\ll 1$. In dimensionless variables, Eqs. (1.1) and (1.2) are written as

$$\delta^2(q_t+qq_\eta-q^2/(2\eta)+wq_z)=-4\eta p_{*\eta}+\delta^2(hh_\eta-h^2/(2\eta)+Hh_z),$$

$$w_t+qw_\eta+ww_z=-p_{*z}+hH_\eta+HH_z,\qquad q_\eta+w_z=0, \tag{1.4}$$

$$a_t+qa_\eta+wa_z=0,\qquad h=-a_z,\qquad H=a_\eta;$$

$$\delta^2h_z^*-4\eta H_\eta^*=0,\qquad æ=1,\qquad H_z^*+h_\eta^*=0. \tag{1.5}$$

Hereinafter, the subscripts indicate the corresponding partial derivative. The boundary conditions (1.3) take the form

$$q = 0, \qquad h = 0 \qquad (\eta = 0), \qquad q = \eta_{1t} + w\eta_{1z} \qquad (\eta = \eta_1),$$
$$p_* = \delta^2 (h^*)^2/(8\eta_1) + 1/(2\eta_1) + (H^*)^2/2 \qquad (\eta = \eta_1), \quad h - H\eta_{1z} = 0 \quad (\eta = \eta_1); \tag{1.6}$$

$$h^* - H^* \eta_{1z} = 0 \qquad (\eta = \eta_1), \quad h^* = 0 \;\; (\eta = \eta_0), \tag{1.7}$$

where $\eta_1(t, z)$ and η_0 correspond to $r_1(t, z)$ and r_0, respectively.

In passing to the long-wave approximation in (1.4)–(1.7), the terms proportional to δ^2 are omitted. In this case, system (1.5) with conditions (1.7) has the solution $h^* = H^*_z(\eta_0 - \eta)$, $H^* = H^*(t, z) = \Phi/(\eta_0 - \eta_1)$, where $\Phi = \text{const}$ is the dimensionless axial flux of the magnetic field between the jet and the cylinder walls. Then, the condition for p_* from (1.6) (with allowance for $\delta^2 \to 0$) acquires the form

$$p_* = 1/(2\eta_1) + \Phi^2/[2(\eta_0 - \eta_1)^2]. \tag{1.8}$$

For system (1.4), the long-wave approximation is not the final one, since it can be further simplified by passing (see [4, 10]) to mixed Euler–Lagrange variables t', z', and ν determined by the relations

$$t = t', \qquad z = z', \qquad \eta = R(t', z', \nu), \qquad \nu \in [0, 1].$$

The function R is assumed to satisfy the equation and the boundary conditions

$$q = R_{t'} + wR_{z'}, \qquad R(t', z', 0) = 0, \qquad R(t', z', 1) = \eta_1(t', z'). \tag{1.9}$$

Thus, the variable ν can be interpreted as the number of the corresponding streamline. In addition, it follows from (1.9) that the boundary conditions (1.6) (for the function q) are satisfied automatically. Note, in the case of this substitution of variables, the unknown free boundary $\eta = \eta_1$ is transformed to the known fixed boundary $\nu = 1$.

In new mixed Euler–Lagrange variables (terms with δ^2 are neglected), Eqs. (1.4) are written as

$$p_{*\nu} = 0,$$
$$R_\nu(w_t + ww_z) = -R_\nu p_{*z} + hH_\nu + R_\nu HH_z - HR_zH_\nu,$$
$$q_\nu + R_\nu w_z - R_z w_\nu = 0, \tag{1.10}$$
$$a_t + wa_z = 0,$$
$$h = -a_z + (R_z/R_\nu)a_\nu, \qquad H = a_\nu/R_\nu.$$

Hereinafter, the primes at the variables t' and z' are omitted.

The boundary conditions (1.6) for the magnetic field in terms of the function a have the form $a_z = 0$ for $\nu = 0$, 1. Equations (1.10) are supplemented by the following initial conditions: $w(0, z, \nu) = w_0(z, \nu)$ and $R(0, z, \nu) = R_0(z, \nu)$; based on the requirements of mutual uniqueness of the transition to mixed Euler–Lagrange variables, $R_0(z, \nu)$ is assumed to be a monotonically increasing function of the argument ν. As the initial condition for the function a, we use $a(0, z, \nu) = a_0(\nu)$. This condition is satisfied if the streamlines are chosen to coincide, at $t = 0$, with the force lines of the magnetic field (because $a = \text{const}$ along the force line). Then, as it follows from the fourth equation of system (1.10), the function $a(t, z, \nu) = a_0(\nu)$ is its solution satisfying the corresponding boundary conditions. Thus, the choice of the function $a(t, z, \nu) = a_0(\nu)$ limits the scope of problems considered only by the requirement of sufficient smoothness of the initial force lines. It is assumed below that $a(t, z, \nu) = a_0(\nu)$.

Using the first equation of system (1.10) and the boundary condition (1.8), we determine p_*. Replacing p_{*z} by the expression obtained for p_* and q by representation (1.9) and assuming that $a = a_0(\nu)$, from (1.10), we obtain the equations

$$w_t + ww_z = \Big(\frac{1}{2R_1^2} - \frac{\Phi^2}{(\eta_0 - R_1)^3}\Big)R_{1z} - \frac{(a_{0\nu})^2}{(R_\nu)^3}\,R_{\nu z},$$
$$(R_\nu)_t + (wR_\nu)_z = 0, \qquad h = R_z a_{0\nu}/R_\nu, \qquad H = a_{0\nu}/R_\nu. \tag{1.11}$$

Here R_1 is the value of the function R for $\nu = 1$; according to the third relation of (1.9), we have $R_1(t, z) \equiv \eta_1(t, z)$.

2. Shear-Free Axial Flow and Uniform Longitudinal Magnetic Field. We consider the class of particular solutions of system (1.11) of the form $w = w(t, z)$, $R = \nu R_1(t, z)$, and $a_0(\nu) = \nu b_0$ (b_0 is a constant). This class of solutions has the following physical interpretation. For $t < 0$, the jet of cylindrical form is located in a uniform magnetic field, the current J passes on the jet surface, and the jet flow is shear-free. At the time $t = 0$, perturbations are introduced into the jet; the magnetic field is assumed to be frozen thereby. Let us study the evolution of these perturbations.

For the class of solutions mentioned, system (1.11) takes the form

$$w_t + ww_z + \Big(\frac{b_0^2}{R_1^3} + \frac{b_1^2(\eta_0 - 1)^2}{(\eta_0 - R_1)^3} - \frac{1}{2R_1^2}\Big)R_{1z} = 0, \tag{2.1}$$

$$R_{1t} + R_1 w_z + wR_{1z} = 0, \qquad h = \nu b_0 R_{1z}/R_1, \qquad H = b_0/R_1.$$

Here $b_1 = \Phi/(\eta_0 - 1)$. From here and from the definition of b_0, it follows that b_0 and b_1 are the ratios of the undisturbed longitudinal fields inside and outside the jet to the characteristic field (i.e., to the azimuthal field at the jet boundary). Thus, this class of solutions is described by a system of two quasilinear equations in partial derivatives. Let us study the type of this system. For this purpose, we find the characteristics on a certain solution (w, R_1).

Note, the system obtained is similar to gas-dynamic equations that describe one-dimensional unsteady isentropic motions of a gas with plane waves [11]. The quantity R_1 has the meaning of density, and the expression

$$c(R_1) = (b_0^2/R_1^2 + b_1^2(\eta_0 - 1)^2 R_1/(\eta_0 - R_1)^3 - 1/(2R_1))^{1/2}$$

has the meaning of the velocity of sound.

The equation of characteristics of system (2.1) has the form $dz/dt = w \pm c$. It follows from here that the system is hyperbolic for $c^2 > 0$ and elliptic for $c^2 < 0$.

We determine the regions of hyperbolicity and ellipticity of the system, since it is of principal importance for understanding the character of the solutions. A clear geometric idea of these regions can be obtained if we introduce the quantities α and β by the relations

$$\alpha = R_1/\eta_0, \qquad \beta = (\eta_0 - R_1)/\eta_0, \qquad 0 \leqslant \alpha < 1, \quad 0 < \beta \leqslant 1.$$

Then, the equality $c^2 = 0$ is satisfied at the points of intersection of two curves in the plane (α, β)

$$\beta = 1 - \alpha, \qquad \beta = \frac{(2b_1^2(\eta_0 - 1)^2)^{1/3}\alpha}{\eta_0^{1/3}(\alpha - 2b_0^2/\eta_0)^{1/3}}, \tag{2.2}$$

located in the domain of definition of α and β. Hence, for $2b_0^2/\eta_0 > 1$, there are no intersection points, and the system is always hyperbolic. For $2b_0^2/\eta_0 = \alpha_0 < 1$, two cases are possible (Fig. 1). Curve 1 in Fig. 1 corresponds to the first equation in (2.2), and curves 2, 3, and 4 correspond to the second equation in (2.2). The system is either always hyperbolic (curve 3) or elliptic–hyperbolic: elliptic for $\alpha_1 < \alpha < \alpha_2$ (curve 4) and hyperbolic outside this interval. Introducing the notation $R_{1*} = \alpha_1\eta_0$ and $R_{2*} = \alpha_2\eta_0$, we obtain that the system is hyperbolic for $0 < R_1 < R_{1*}$ and $R_1 > R_{2*}$ and elliptic for $R_{1*} < R_1 < R_{2*}$. Note, the condition of hyperbolicity on the steady solution corresponding to the shear-free flow in a cylindrical jet of constant radius $R_1 = 1$ is $b_0^2 + b_1^2/(\eta_0 - 1) - 1/2 > 0$, i.e., coincides with the necessary and sufficient condition of linear stability of such a flow, obtained by the spectral method [1].

An essential factor for analytical studies is the fact that, in the region of hyperbolicity, the Riemann invariants, i.e., the functions $s(w, R_1)$ and $l(w, R_1)$, which remain constant on the characteristics, are found in an explicit form:

$$s = w - \sigma(R_1), \qquad l = w + \sigma(R_1), \qquad \sigma(R_1) = \int\limits_{R_1}^{R_*} \frac{c(\xi)}{\xi}\, d\xi. \tag{2.3}$$

Here R_* is a certain constant from the interval $(0, \eta_0)$ if the system is hyperbolic and $R_* = R_{1*}$ if the system is of the mixed type [in the second case, only the interval $(0, R_{1*})$ is considered in the region of hyperbolicity]. The invariants s and l are conserved along the characteristics described by the equations $dz/dt = w + c$ and $dz/dt = w - c$, respectively [i.e., $s_t + (w + c)s_z = 0$ and $l_t + (w - c)l_z = 0$]. By analogy with gas dynamics, following [6], the plane of variables (w, R_1) is called the hodograph plane. In studying hyperbolic systems, the graphical representation

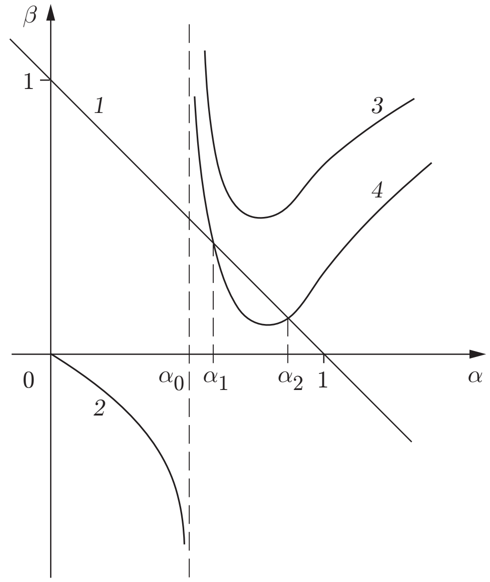

Fig. 1

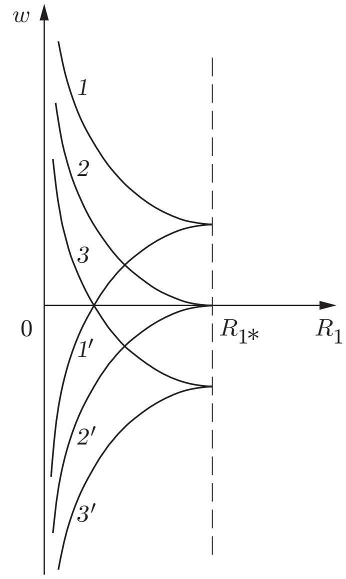

Fig. 2

of characteristics on the hodograph plane is used. Let us consider the case where the region of ellipticity exists. The behavior of characteristics on the hodograph plane in the band $0 < R_1 < R_{1*}$ is shown in Fig. 2. Curves 1–3 correspond to the line $s = \text{const}$, and curves $1'$–$3'$ correspond to the line $l = \text{const}$ for different values of constants. In passing through the line $R_1 = R_{1*}$ (transitional line), the type of the system changes. Note, in contrast to [6], where the characteristics on transitional lines have both second-order tangency points and cuspidal points, there are only cuspidal points on the transitional line $R_1 = R_{1*}$. The behavior of characteristics near the transitional line is important for studying the question whether the solution can leave the region of hyperbolicity in the course of evolution [12].

We consider the case $\eta_0 \gg 1$, where the radius of the external cylinder is much greater than the jet radius. Passing to the limit $\eta_0 \to \infty$, we obtain

$$c = [b_0^2/R_1^2 - 1/(2R_1)]^{1/2}. \tag{2.4}$$

It follows from here that the system in this case is a system of the mixed type: it is hyperbolic in the band $0 < R_1 < 2b_0^2$ and elliptic in the region $R_1 > 2b_0^2$.

3. Collision of Magnetized Jets. We use the theory described in Sec. 2 to solving a particular problem. First, we perform a mathematical study and then discuss the possibility of physical application. To simplify the mathematical manipulations, we consider the case $\eta_0 \gg 1$, i.e., we take the dependence $c(R_1)$ in the form (2.4). Such simplification retains the main features of the equations examined.

At the time $t = 0$, the following perturbation is introduced into an initially motionless cylindrical jet of unit radius: the fluid located in the region $z < 0$ acquires a velocity w_1, and the fluid located in the region $z > 0$ acquires a velocity w_2 ($w_1 > w_2$). We assume that the initial conditions belong to the region of hyperbolicity, i.e., the inequality $2b_0^2 > 1$ is satisfied. We determine for which values of w_1 and w_2 continuous motion is possible at subsequent times ($t > 0$) and find it.

We seek the solution in the form of a combination of simple centered waves and motions with a constant velocity. By virtue of the above-mentioned analogy between Eqs. (2.1) and gas-dynamic equations, the properties of simple waves are also similar. We assume that l- and s-waves are simple waves in which the invariants l and s are constant (then, the characteristics along which the invariants s and l are constant are straight lines). We determine how the tangency of the slope of characteristics in the plane (z, t) in l- and s-waves changes with varied radius of the jet R_1. We consider the l-wave. We assume that $k_1 = w + c$ and $l = l_0 = w + \sigma(R_1)$. Hence, $k_1 = l_0 + c(R_1) - \sigma(R_1)$. Differentiating k_1 with respect to R_1 and taking into account expressions (2.3) and (2.4) for σ and c, we obtain

$$\frac{dk_1}{dR_1} = \frac{dc}{dR_1} + \frac{c}{R_1} = -\frac{2^{1/2}}{4(2b_0^2 - R_1)^{1/2}} < 0. \tag{3.1}$$

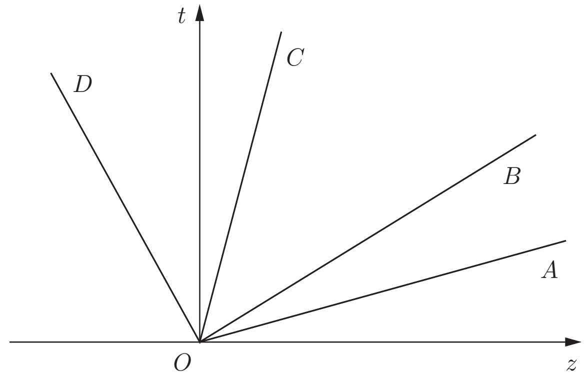

Fig. 3

For the s-wave, we assume that $k_2 = w - c$ and $s = s_0 = w - \sigma(R_1)$. Therefore, we obtain

$$\frac{dk_2}{dR_1} = -\frac{dk_1}{dR_1} > 0. \tag{3.2}$$

The flow pattern in the plane (z, t) with allowance for (3.1) and (3.2) is presented in Fig. 3. The sectors AOB and COD contain the l- and s-waves with $l_0 = w_2 + \sigma(1)$ and $s_0 = w_1 - \sigma(1)$, respectively. Along the ray OA ($z = [w_2 + c(1)]t$), the l-wave is adjacent to the flow region with constant w and R_1: $w = w_2$ and $R_1 = 1$. Along the rays OB and OC, the l- and s-waves are also adjacent to the flow region with constant w and R_1: $w = w_3$ and $R_1 = R_{13}$. The equation of the ray OB is $z = [w_3 + c(R_{13})]t$, and the equation of the ray OC is $z = [w_3 - c(R_{13})]t$. Along the ray OD ($z = [w_1 - c(1)]t$), the s-wave is adjacent to the flow region with constant w and R_1: $w = w_1$ and $R_1 = 1$. Because the region of hyperbolicity is located to the left of the straight line $R_1 = R_{1*}$ (in the case considered, $R_{1*} = 2b_0^2$), the motion corresponding to Fig. 3 is possible not for all values of w_1 and w_2. Since the l- and s-waves are adjacent to the same flow region with $w = w_3$ and $R_1 = R_{13}$, the graphs of the l_0- and s_0-waves on the hodograph plane (see Fig. 2) should intersect at the point (w_3, R_{13}). Obviously, this is possible only for $l_0 > s_0$. Substituting the expressions for l_0 and s_0, we find that this kind of motion can be realized only in the case

$$w_1 - w_2 \leqslant 2\sigma(1) = 2\int\limits_1^{2b_0^2} \frac{c(\xi)}{\xi}\, d\xi,$$

i.e., if the difference in velocities is not very large. For $w_1 - w_2 = 2\sigma(1)$, the constant solution (w_3, R_{13}) falls onto the interface between the hyperbolicity and ellipticity regions. We determine the values of w_3 and R_{13}. For the l_0- and s_0-waves, we obtain $l_0 = w_3 + \sigma(R_{13})$ and $s_0 = w_3 - \sigma(R_{13})$. Substituting the expressions for l_0 and s_0 into these equalities, we obtain

$$w_3 = (w_1 + w_2)/2, \qquad \sigma(R_{13}) = \sigma(1) - (w_1 - w_2)/2.$$

The latter expression serves to find R_{13}. It follows from inequalities (3.1) and (3.2) that $R_{13} > 1$. Thus, bulges with relative velocities $c(1)$ and $-c(1)$ propagate into the regions with constant velocities and radii $(w_1, 1)$ and $(w_2, 1)$, respectively.

The results obtained are generalized to the case where the fluid has different densities for $z < 0$ and $z > 0$: ρ_1 and ρ_2. If we use ρ_2 as the normalizing density, then the factor ρ_1/ρ_2 appears at convective terms in the equations for the fluid of density ρ_1. Then, the "velocity of sound" c and the function σ for this fluid for identical values of R_1 are $(\rho_1/\rho_2)^{1/2}$ times lower than the corresponding values for the fluid of density ρ_2. As a result, w_3 and R_{13} are found from the relations

$$w_3 = \frac{w_1(\rho_1/\rho_2)^{1/2} + w_2}{1 + (\rho_1/\rho_2)^{1/2}}, \qquad \sigma(R_{13}) = \sigma(1) - \frac{(w_1 - w_2)(\rho_1/\rho_2)^{1/2}}{1 + (\rho_1/\rho_2)^{1/2}}.$$

It follows from the latter equality than the solution is possible if

$$w_1 - w_2 \leqslant \sigma(1)[1 + (\rho_1/\rho_2)^{1/2}]/(\rho_1/\rho_2)^{1/2}.$$

In addition, between the rays OB and OC, there appears the ray $z = w_3 t$ corresponding to the contact discontinuity between the fluids with densities ρ_1 and ρ_2.

This solution can be used to analyze the collision of magnetized jets. It describes the propagation of perturbations over the jets and yields the critical value for the velocity difference before which there exists a continuous self-similar solution. If the difference exceeds this critical value, new effects can appear, for instance, the jets can be destroyed.

Conclusions. Thus, a model is constructed, which describes the nonlinear evolution of long-wave axisymmetric perturbations on a conducting fluid jet with surface electric current, located along the axis of a conducting solid cylinder in a longitudinal magnetic field. The possibility of using the model for analytical studies is demonstrated. The special feature of the model system of equations is that this system can be either hyperbolic or elliptic-hyperbolic if the longitudinal field is uniform and the axial flow is shear-free. In the region of hyperbolicity, the Riemann invariants are calculated in an explicit form, which is important for analytical studies. The possibility of solution transition from the hyperbolic region to the elliptic region can mean, for instance, that the stability of a certain steady solution in the linear approximation (which corresponds to hyperbolicity of the system on the given solution) does not guarantee that instability does not appear at the nonlinear stage of perturbation development (when the system passes to the region of ellipticity). Possibly, this is one of the reasons that conductors with electric current passing over them in the longitudinal magnetic field are destroyed even if the magnitude of the longitudinal field is much greater than the threshold of linear stability [13]. The problem of decay of the velocity discontinuity on the jet is solved. We believe that this solution can be used to analyze the collision of magnetized jets.

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